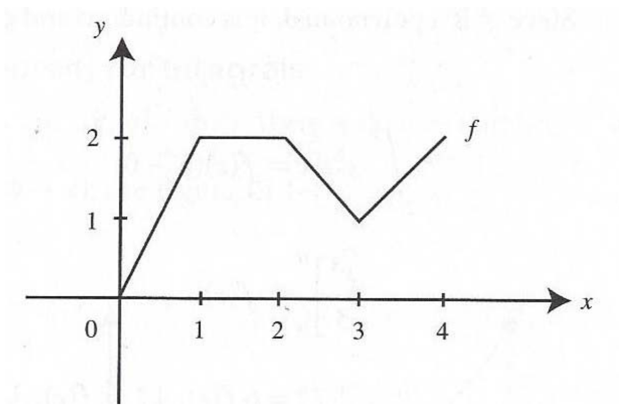
Applications of Differential Equations BC:

Average Value of a function $f(x)$ on an interval $[a, b]$:

MVT for Integrals:

EX: Given $f(x) = \sqrt{x - 1}$, verify MVT for Integrals for f on [1,10], and find the value of c as indicated by the thrm.

EX: The graph is given, find the average value of f on [0,4].



EX: The velocity of a particle moving along a line is $v(t) = 3t^2 - 18t + 24$. Find the average velocity from $t = 1$ to $t = 3$.

EX: On a certain day, the changes in the temperature in a greenhouse beginning at 12 noon are represented by $f(t) = sin\left(\frac{t}{2}\right)$ degrees Fahrenheit, where t is the number of hours elapsed after 12 noon. If at 12 noon, the temperature is 95°F, find the temperature in the greenhouse at 5 pm.

EX: Water is leaking from a faucet at the rate of $l(t) = 10e^{-0.5t}$ gallons per hour, where t is measured in hours. How many gallons of water will have leaked from the faucet after a 24 hour period?

EX: Write an equation for the curve that passes through the point (3, 4) and has a slope at any point (x, y) as $\frac{dy}{dx} = \frac{x^2+1}{2y}$.

EX: Determine the particular solution for $\frac{dy}{dx} = 4x^3y^2$ given that $y(1) = -\frac{1}{2}$.

Growth & Decay Models

1. If $\frac{dy}{dt} = ky$, then the rate of change y is proportional to y.

2. If y is a differentiable function of t with $y > 0$, $\frac{dy}{dt} = ky$ then $y(t) = Ce^{kt}$; where C is initial value of y and k is proportionality constant. If $k > 0$, then k is a growth constant and if $k < 0$, then k is the decay constant.

EX: If the amount of bacteria in a culture at any time increases at a rate proportional to the amount of bacteria present and there are 500 bacteria after one day and 800 bacteria after the third day:

1) approximately how many bacteria are there initially, and

2) approximately how many bacteria are there after 4 days?

EX: Radioactive Radium has a half-life of approximately 1599 years. What amount remains after 1000 years if the initial quantity was 10 g?

EX: The rate of change of the number of coyotes $N(t)$ in a population is directly proportional to $650 - N(t)$, where t is the time in years. When $t = 0$, the population is 300, and when $t = 2$, the population has increased to 500. Find the population when $t = 3$.

EX: During a chemical reaction, substance A is converted into substance B at a rate that is proportional to the square of the amount of A. When $t = 0$, 60 g of A is present, and after 1 hour ($t = 1$), only 10 g of A remains unconverted. How much of A is present after 2 hours?