Applied Differential Equations Day 2

1. Under ideal conditions, air pressure decreases continuously with the height above sea level at a rate proportional to the pressure at that height. The barometer reads 30 inches at sea level and 15 inches at 18,000 feet. Find the barometric pressure at 35,000 feet.

2. Radioactive radium has a half-life of approximately 1599 years. The initial quantity is 15 grams. How much remains after 750 years?

3. At any time $t \geq 0$ in hours, the rate of growth of a population of bacteria is given by $\dfrac{dy}{dt} = \dfrac{1}{2}y$. Initially, there are 200 bacteria in the culture.
    a. Use separation of variables to solve y, the number of bacteria present, at any time $t \geq 0$.
    b. Write, but do not evaluate an expression to find the average number of bacteria in the population for $0 \leq t \leq 10$.
    c. Write an expression to find the average rate of bacteria growth over the first 10 hours of growth. Indicate units of measure.

4. Given the differential equation $y' = \dfrac{2x}{y}$ with a particular solution in the form of $y = f(x)$ that satisfies the initial condition $f(1) = 2$:
    a. Use Euler's Method, starting at $x = 1$ with two steps of equal size, to approximate $y(1.4)$. Show the work that leads to your answer.
    b. Find the particular solution to the given differential equation that passes through $(1,2)$ and state its domain.

5. If $\dfrac{dy}{dx} = 2xy^2$, and $y(-1) = 2$, find $y(2)$.

6. When an object is removed from a furnace and placed in an environment with a constant temperature of $80°F$, its core temperature is $1500°F$. One hour after it is removed, the core temperature is $1120°F$. Find the core temperature 5 hours after the object is removed from the furnace.

7. The management at a certain factory has found that a worker can produce at most 30 units in a day. The learning curve for the number of units N produced per day after a new employee has worked t days is $N = 30(1 - e^{kt})$. After 20 days on the job, a particular worker produces 19 unites.
    a. Find the learning curve for this worker
    b. How many days should pass before this worker is producing 25 units per day?