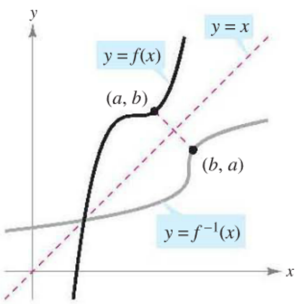
3.6 Derivatives of Inverse Functions

- Find the derivative of an inverse function.
- Differentiate an inverse trigonometric function.
- Review the basic differentiation rules for elementary functions.

Derivative of an Inverse Function

The next two theorems discuss the derivative of an inverse function. The reasonableness of Theorem 3.16 follows from the reflective property of inverse functions, as shown in Figure 3.33. Proofs of the two theorems are given in Appendix A.



The graph of f^{-1} is a reflection of the graph of f in the line $y = x$.
Figure 3.33

THEOREM 3.16 CONTINUITY AND DIFFERENTIABILITY OF INVERSE FUNCTIONS

Let f be a function whose domain is an interval I. If f has an inverse function, then the following statements are true.

1. If f is continuous on its domain, then f^{-1} is continuous on its domain.
2. If f is differentiable on an interval containing c and $f'(c) \neq 0$, then f^{-1} is differentiable at $f(c)$.

THEOREM 3.17 THE DERIVATIVE OF AN INVERSE FUNCTION

Let f be a function that is differentiable on an interval I. If f has an inverse function g, then g is differentiable at any x for which $f'(g(x)) \neq 0$. Moreover,

$$g'(x) = \frac{1}{f'(g(x))}, \quad f'(g(x)) \neq 0.$$

EXAMPLE 1 Evaluating the Derivative of an Inverse Function

Let $f(x) = \frac{1}{4}x^3 + x - 1$.

a. What is the value of $f^{-1}(x)$ when $x = 3$?

b. What is the value of $(f^{-1})'(x)$ when $x = 3$?

Solution Notice that f is one-to-one and therefore has an inverse function.

a. Because $f(2) = 3$, you know that $f^{-1}(3) = 2$.

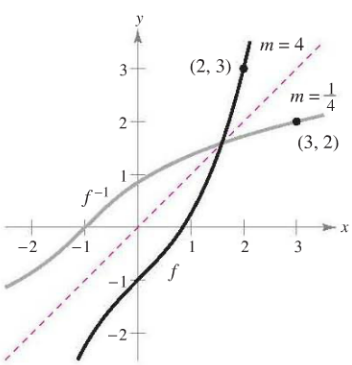
b. Because the function f is differentiable and has an inverse function, you can apply Theorem 3.17 (with $g = f^{-1}$) to write

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(2)}.$$

Moreover, using $f'(x) = \frac{3}{4}x^2 + 1$, you can conclude that

$$\begin{aligned}(f^{-1})'(3) &= \frac{1}{f'(2)}\\ &= \frac{1}{\frac{3}{4}(2^2) + 1}\\ &= \frac{1}{4}.\end{aligned}$$

(See Figure 3.34.)



The graphs of the inverse functions f and f^{-1} have reciprocal slopes at points (a, b) and (b, a).
Figure 3.34

In Example 1, note that at the point $(2, 3)$ the slope of the graph of f is 4 and at the point $(3, 2)$ the slope of the graph of f^{-1} is $\frac{1}{4}$ (see Figure 3.34). This reciprocal relationship (which follows from Theorem 3.17) is sometimes written as

$$\frac{dy}{dx} = \frac{1}{dx/dy}.$$

EXAMPLE 2 Graphs of Inverse Functions Have Reciprocal Slopes

Let $f(x) = x^2$ (for $x \geq 0$) and let $f^{-1}(x) = \sqrt{x}$. Show that the slopes of the graphs of f and f^{-1} are reciprocals at each of the following points.

a. $(2, 4)$ and $(4, 2)$ **b.** $(3, 9)$ and $(9, 3)$

Solution The derivatives of f and f^{-1} are $f'(x) = 2x$ and $(f^{-1})'(x) = \dfrac{1}{2\sqrt{x}}$.

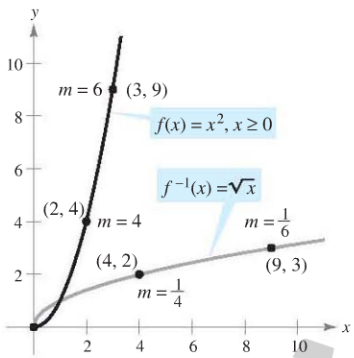
a. At $(2, 4)$, the slope of the graph of f is $f'(2) = 2(2) = 4$. At $(4, 2)$, the slope of the graph of f^{-1} is

$$(f^{-1})'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2(2)} = \frac{1}{4}.$$

b. At $(3, 9)$, the slope of the graph of f is $f'(3) = 2(3) = 6$. At $(9, 3)$, the slope of the graph of f^{-1} is

$$(f^{-1})'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{2(3)} = \frac{1}{6}.$$

So, in both cases, the slopes are reciprocals, as shown in Figure 3.35. ■



At $(0, 0)$, the derivative of f is 0 and the derivative of f^{-1} does not exist.
Figure 3.35

When determining the derivative of an inverse function, you have two options: (1) you can apply Theorem 3.17, or (2) you can use implicit differentiation. The first approach is illustrated in Example 3, and the second in the proof of Theorem 3.18.

EXAMPLE 3 Finding the Derivative of an Inverse Function

Find the derivative of the inverse tangent function.

Solution Let $f(x) = \tan x$, $-\pi/2 < x < \pi/2$. Then let $g(x) = \arctan x$ be the inverse tangent function. To find the derivative of $g(x)$, use the fact that $f'(x) = \sec^2 x = \tan^2 x + 1$, and apply Theorem 3.17 as follows.

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{f'(\arctan x)} = \frac{1}{[\tan(\arctan x)]^2 + 1} = \frac{1}{x^2 + 1}$$ ■

Derivatives of Inverse Trigonometric Functions

In Section 3.4, you saw that the derivative of the *transcendental* function $f(x) = \ln x$ is the *algebraic* function $f'(x) = 1/x$. You will now see that the derivatives of the inverse trigonometric functions also are algebraic (even though the inverse trigonometric functions are themselves transcendental).

The following theorem lists the derivatives of the six inverse trigonometric functions. Note that the derivatives of arccos u, arccot u, and arccsc u are the *negatives* of the derivatives of arcsin u, arctan u, and arcsec u, respectively.

THEOREM 3.18 DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x.

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1+u^2}$$

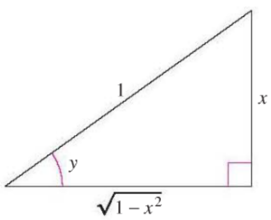
$$\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

PROOF Let $y = \arcsin x$, $-\pi/2 \le y \le \pi/2$ (see Figure 3.36). So, $\sin y = x$, and you can use implicit differentiation as follows.

$$\sin y = x$$

$$(\cos y)\left(\frac{dy}{dx}\right) = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$$



$y = \arcsin x$
Figure 3.36

Because u is a differentiable function of x, you can use the Chain Rule to write

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}, \quad \text{where} \quad u' = \frac{du}{dx}.$$

Proofs of the other differentiation rules are left as an exercise (see Exercise 73). ■

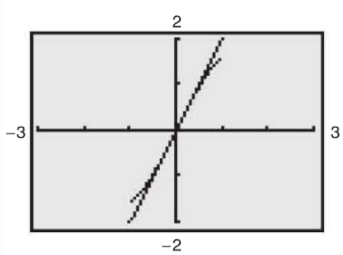
There is no common agreement on the definition of arcsec x (or arccsc x) for negative values of x. When we defined the range of the arcsecant, we chose to preserve the reciprocal identity $\text{arcsec } x = \arccos(1/x)$. For example, to evaluate $\text{arcsec}(-2)$, you can write

$$\text{arcsec}(-2) = \arccos(-0.5)$$
$$\approx 2.09.$$

One of the consequences of the definition of the inverse secant function given in this text is that its graph has a positive slope at every x-value in its domain. This accounts for the absolute value sign in the formula for the derivative of arcsec x.

EXPLORATION

Suppose that you want to find a linear approximation to the graph of the function in Example 4. You decide to use the tangent line at the origin, as shown below. Use a graphing utility to describe an interval about the origin where the tangent line is within 0.01 unit of the graph of the function. What might a person mean by saying that the original function is "locally linear"?



EXAMPLE 4 A Derivative That Can Be Simplified

Differentiate $y = \arcsin x + x\sqrt{1-x^2}$.

Solution

$$y' = \frac{1}{\sqrt{1-x^2}} + x\left(\frac{1}{2}\right)(-2x)(1-x^2)^{-1/2} + \sqrt{1-x^2}$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2}$$

$$= \sqrt{1-x^2} + \sqrt{1-x^2}$$

$$= 2\sqrt{1-x^2}$$ ■

EXAMPLE 5 Differentiating Inverse Trigonometric Functions

a. $$\frac{d}{dx}[\arcsin(2x)] = \frac{2}{\sqrt{1-(2x)^2}} \qquad u = 2x$$
$$= \frac{2}{\sqrt{1-4x^2}}$$

b. $$\frac{d}{dx}[\arctan(3x)] = \frac{3}{1+(3x)^2} \qquad u = 3x$$
$$= \frac{3}{1+9x^2}$$

c. $$\frac{d}{dx}[\arcsin\sqrt{x}] = \frac{(1/2)x^{-1/2}}{\sqrt{1-x}} \qquad u = \sqrt{x}$$
$$= \frac{1}{2\sqrt{x}\sqrt{1-x}}$$
$$= \frac{1}{2\sqrt{x-x^2}}$$

d. $$\frac{d}{dx}[\operatorname{arcsec} e^{2x}] = \frac{2e^{2x}}{e^{2x}\sqrt{(e^{2x})^2-1}} \qquad u = e^{2x}$$
$$= \frac{2e^{2x}}{e^{2x}\sqrt{e^{4x}-1}}$$
$$= \frac{2}{\sqrt{e^{4x}-1}}$$

In part (d), the absolute value sign is not necessary because $e^{2x} > 0$. ■

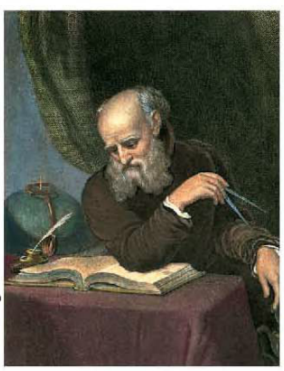
Review of Basic Differentiation Rules

In the 1600s, Europe was ushered into the scientific age by such great thinkers as Descartes, Galileo, Huygens, Newton, and Kepler. These men believed that nature is governed by basic laws—laws that can, for the most part, be written in terms of mathematical equations. One of the most influential publications of this period—*Dialogue on the Great World Systems*, by Galileo Galilei—has become a classic description of modern scientific thought.

As mathematics has developed during the past few hundred years, a small number of elementary functions has proven sufficient for modeling most* phenomena in physics, chemistry, biology, engineering, economics, and a variety of other fields. An **elementary function** is a function from the following list or one that can be formed as the sum, product, quotient, or composition of functions in the list.

Algebraic Functions	*Transcendental Functions*
Polynomial functions	Logarithmic functions
Rational functions	Exponential functions
Functions involving radicals	Trigonometric functions
	Inverse trigonometric functions

With the differentiation rules introduced so far in the text, you can differentiate any elementary function. For convenience, these differentiation rules are summarized on the next page.


The Granger Collection

GALILEO GALILEI (1564–1642)

Galileo's approach to science departed from the accepted Aristotelian view that nature had describable *qualities*, such as "fluidity" and "potentiality." He chose to describe the physical world in terms of measurable *quantities*, such as time, distance, force, and mass.

* *Some important functions used in engineering and science (such as Bessel functions and gamma functions) are not elementary functions.*

BASIC DIFFERENTIATION RULES FOR ELEMENTARY FUNCTIONS

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
22. $\frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1 + u^2}$
23. $\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
24. $\frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$

3.6 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–8, verify that f has an inverse. Then use the function f and the given real number a to find $(f^{-1})'(a)$. (*Hint:* See Example 1.)

Function	*Real Number*
1. $f(x) = x^3 - 1$	$a = 26$
2. $f(x) = 5 - 2x^3$	$a = 7$
3. $f(x) = x^3 + 2x - 1$	$a = 2$
4. $f(x) = \frac{1}{27}(x^5 + 2x^3)$	$a = -11$
5. $f(x) = \sin x, \ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$a = \frac{1}{2}$
6. $f(x) = \cos 2x, \ 0 \le x \le \frac{\pi}{2}$	$a = 1$
7. $f(x) = \frac{x + 6}{x - 2}, \ x > 2$	$a = 3$
8. $f(x) = \sqrt{x - 4}$	$a = 2$

In Exercises 9–12, show that the slopes of the graphs of f and f^{-1} are reciprocals at the given points.

Function	*Point*
9. $f(x) = x^3$	$\left(\frac{1}{2}, \frac{1}{8}\right)$
$f^{-1}(x) = \sqrt[3]{x}$	$\left(\frac{1}{8}, \frac{1}{2}\right)$
10. $f(x) = 3 - 4x$	$(1, -1)$
$f^{-1}(x) = \frac{3 - x}{4}$	$(-1, 1)$
11. $f(x) = \sqrt{x - 4}$	$(5, 1)$
$f^{-1}(x) = x^2 + 4, \quad x \ge 0$	$(1, 5)$
12. $f(x) = \frac{4}{1 + x^2}, \quad x \ge 0$	$(1, 2)$
$f^{-1}(x) = \sqrt{\frac{4 - x}{x}}$	$(2, 1)$

In Exercises 13–16, (a) find an equation of the tangent line to the graph of f at the given point and (b) use a graphing utility to graph the function and its tangent line at the point.

Function	*Point*
13. $f(x) = \arccos x^2$	$\left(0, \frac{\pi}{2}\right)$
14. $f(x) = \arctan x$	$\left(-1, -\frac{\pi}{4}\right)$
15. $f(x) = \arcsin 3x$	$\left(\frac{\sqrt{2}}{6}, \frac{\pi}{4}\right)$
16. $f(x) = \text{arcsec } x$	$\left(\sqrt{2}, \frac{\pi}{4}\right)$

In Exercises 17–20, find dy/dx at the given point for the equation.

17. $x = y^3 - 7y^2 + 2, \ (-4, 1)$

18. $x = 2\ln(y^2 - 3), \ (0, 2)$

19. $x \arctan x = e^y, \ \left(1, \ln\frac{\pi}{4}\right)$

20. $\arcsin xy = \frac{2}{3}\arctan 2x, \ \left(\frac{1}{2}, 1\right)$

In Exercises 21–46, find the derivative of the function.

21. $f(x) = \arcsin(x + 1)$

22. $f(t) = \arcsin t^2$

23. $g(x) = 3 \arccos \dfrac{x}{2}$

24. $f(x) = \operatorname{arcsec} 4x$

25. $f(x) = \arctan e^x$

26. $f(x) = \operatorname{arccot} \sqrt{2x}$

27. $g(x) = \dfrac{\arcsin 3x}{x}$

28. $h(x) = x^2 \arctan 5x$

29. $g(x) = \dfrac{\arccos x}{x + 1}$

30. $g(x) = e^{2x} \arcsin x$

31. $h(x) = \operatorname{arccot} 6x$

32. $f(x) = \operatorname{arccsc} 3x$

33. $h(t) = \sin(\arccos t)$

34. $f(x) = \arcsin x + \arccos x$

35. $y = 2x \arccos x - 2\sqrt{1 - x^2}$

36. $y = \ln(t^2 + 4) - \dfrac{1}{2} \arctan \dfrac{t}{2}$

37. $y = \dfrac{1}{2}\left(\dfrac{1}{2} \ln \dfrac{x + 1}{x - 1} + \arctan x\right)$

38. $y = \dfrac{1}{2}\left[x\sqrt{4 - x^2} + 4 \arcsin\left(\dfrac{x}{2}\right)\right]$

39. $g(t) = \tan(\arcsin t)$

40. $f(x) = \operatorname{arcsec} x + \operatorname{arccsc} x$

41. $y = x \arcsin x + \sqrt{1 - x^2}$

42. $y = x \arctan 2x - \frac{1}{4} \ln(1 + 4x^2)$

43. $y = 8 \arcsin \dfrac{x}{4} - \dfrac{x\sqrt{16 - x^2}}{2}$

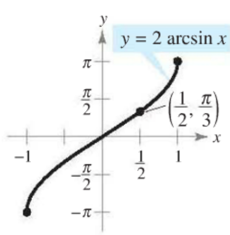
44. $y = 25 \arcsin \dfrac{x}{5} - x\sqrt{25 - x^2}$

45. $y = \arctan x + \dfrac{x}{1 + x^2}$

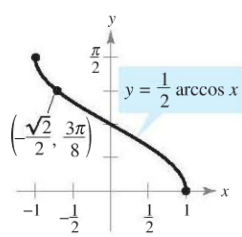
46. $y = \arctan \dfrac{x}{2} - \dfrac{1}{2(x^2 + 4)}$

In Exercises 47–52, find an equation of the tangent line to the graph of the function at the given point.

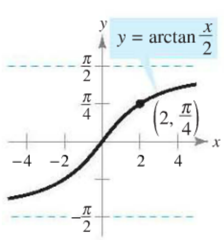
47. $y = 2 \arcsin x$



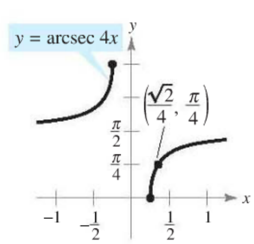
48. $y = \dfrac{1}{2} \arccos x$



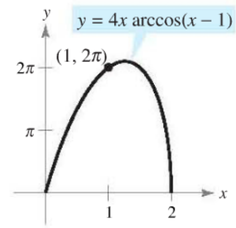
49. $y = \arctan \dfrac{x}{2}$



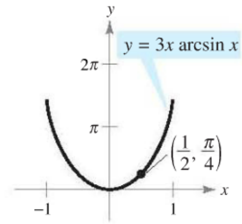
50. $y = \operatorname{arcsec} 4x$



51. $y = 4x \arccos(x - 1)$



52. $y = 3x \arcsin x$



53. Find equations of all tangent lines to the graph of $f(x) = \arccos x$ that have slope -2.

54. Find an equation of the tangent line to the graph of $g(x) = \arctan x$ when $x = 1$.

CAS ***Linear and Quadratic Approximations*** **In Exercises 55–58, use a computer algebra system to find the linear approximation**

$$P_1(x) = f(a) + f'(a)(x - a)$$

and the quadratic approximation

$$P_2(x) = f(a) + f'(a)(x - a) + \tfrac{1}{2}f''(a)(x - a)^2$$

to the function f at $x = a$. Sketch the graph of the function and its linear and quadratic approximations.

55. $f(x) = \arcsin x, \quad a = \frac{1}{2}$

56. $f(x) = \arctan x, \quad a = 1$

57. $f(x) = \arctan x, \quad a = 0$

58. $f(x) = \arccos x, \quad a = 0$

Implicit Differentiation **In Exercises 59–62, find an equation of the tangent line to the graph of the equation at the given point.**

59. $x^2 + x \arctan y = y - 1, \quad \left(-\dfrac{\pi}{4}, 1\right)$

60. $\arctan(xy) = \arcsin(x + y), \quad (0, 0)$

61. $\arcsin x + \arcsin y = \dfrac{\pi}{2}, \quad \left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$

62. $\arctan(x + y) = y^2 + \dfrac{\pi}{4}, \quad (1, 0)$

WRITING ABOUT CONCEPTS

In Exercises 63 and 64, the derivative of the function has the same sign for all x in its domain, but the function is not one-to-one. Explain.

63. $f(x) = \tan x$

64. $f(x) = \dfrac{x}{x^2 - 4}$

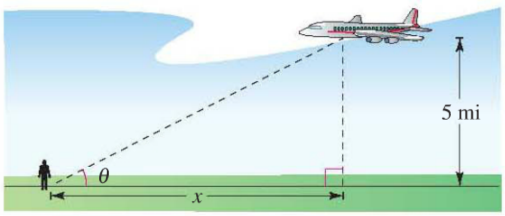
65. State the theorem that gives the method for finding the derivative of an inverse function.

66. Are the derivatives of the inverse trigonometric functions algebraic or transcendental functions? List the derivatives of the inverse trigonometric functions.

67. ***Angular Rate of Change*** An airplane flies at an altitude of 5 miles toward a point directly over an observer. Consider θ and x as shown in the figure.

(a) Write θ as a function of x.

(b) The speed of the plane is 400 miles per hour. Find $d\theta/dt$ when $x = 10$ miles and $x = 3$ miles.



68. ***Writing*** Repeat Exercise 67 if the altitude of the plane is 3 miles and describe how the altitude affects the rate of change of θ.

69. ***Angular Rate of Change*** In a free-fall experiment, an object is dropped from a height of 256 feet. A camera on the ground 500 feet from the point of impact records the fall of the object (see figure).

(a) Find the position function giving the height of the object at time t, assuming the object is released at time $t = 0$. At what time will the object reach ground level?

(b) Find the rates of change of the angle of elevation of the camera when $t = 1$ and $t = 2$.

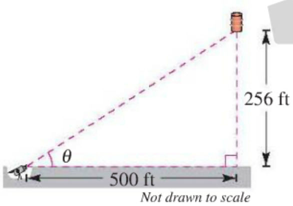


Figure for 69

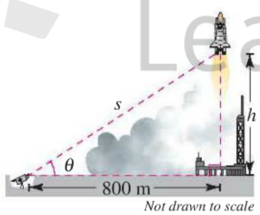


Figure for 70

70. ***Angular Rate of Change*** A television camera at ground level is filming the lift-off of a space shuttle at a point 800 meters from the launch pad. Let θ be the angle of elevation of the shuttle and let s be the distance between the camera and the shuttle (see figure). Write θ as a function of s for the period of time when the shuttle is moving vertically. Differentiate the result to find $d\theta/dt$ in terms of s and ds/dt.

71. ***Angular Rate of Change*** An observer is standing 300 feet from the point at which a balloon is released. The balloon rises at a rate of 5 feet per second. How fast is the angle of elevation of the observer's line of sight increasing when the balloon is 100 feet high?

72. ***Angular Speed*** A patrol car is parked 50 feet from a long warehouse (see figure). The revolving light on top of the car turns at a rate of 30 revolutions per minute. Write θ as a function of x. How fast is the light beam moving along the wall when the beam makes an angle of $\theta = 45°$ with the line perpendicular from the light to the wall?

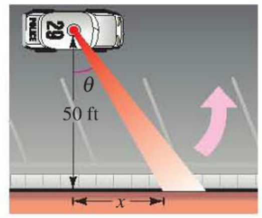


Figure for 72

73. Verify each differentiation formula.

(a) $\dfrac{d}{dx}[\arccos u] = \dfrac{-u'}{\sqrt{1 - u^2}}$

(b) $\dfrac{d}{dx}[\arctan u] = \dfrac{u'}{1 + u^2}$

(c) $\dfrac{d}{dx}[\operatorname{arcsec} u] = \dfrac{u'}{|u|\sqrt{u^2 - 1}}$

(d) $\dfrac{d}{dx}[\operatorname{arccot} u] = \dfrac{-u'}{1 + u^2}$

(e) $\dfrac{d}{dx}[\operatorname{arccsc} u] = \dfrac{-u'}{|u|\sqrt{u^2 - 1}}$

74. ***Existence of an Inverse*** Determine the values of k such that the function $f(x) = kx + \sin x$ has an inverse function.

True or False? **In Exercises 75 and 76, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

75. The slope of the graph of the inverse tangent function is positive for all x.

76. $\dfrac{d}{dx}[\arctan(\tan x)] = 1$ for all x in the domain.

77. Prove that $\arcsin x = \arctan\left(\dfrac{x}{\sqrt{1 - x^2}}\right)$, $|x| < 1$.

78. Prove that $\arccos x = \dfrac{\pi}{2} - \arctan\left(\dfrac{x}{\sqrt{1 - x^2}}\right)$, $|x| < 1$.

79. Some calculus textbooks define the inverse secant function using the range $[0, \pi/2) \cup [\pi, 3\pi/2)$.

(a) Sketch the graph of $y = \operatorname{arcsec} x$ using this range.

(b) Show that $y' = \dfrac{1}{x\sqrt{x^2 - 1}}$.

80. Compare the graphs of $y_1 = \sin(\arcsin x)$ and $y_2 = \arcsin(\sin x)$. What are the domains and ranges of y_1 and y_2?

81. Show that the function $f(x) = \arcsin \dfrac{x - 2}{2} - 2 \arcsin\left(\dfrac{\sqrt{x}}{2}\right)$ is constant for $0 \le x \le 4$.

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82. ***Think About It*** The point $(1, 3)$ lies on the graph of f, and the slope of the tangent line through this point is $m = 2$. Assume f^{-1} exists. What is the slope of the tangent line to the graph of f^{-1} at the point $(3, 1)$?